Quadratic Equations Exercise A, Question 1

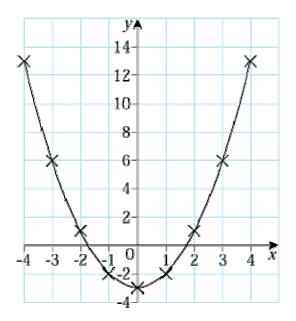
Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = x^2 - 3$

Solution:

 $y = x^2 - 3$.



Equation of line of symmetry is x = 0.

Quadratic Equations Exercise A, Question 2

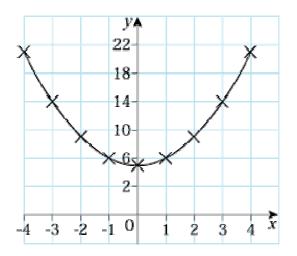
Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = x^2 + 5$

Solution:

 $y = x^2 + 5.$



Equation of line of symmetry is x = 0.

Quadratic Equations Exercise A, Question 3

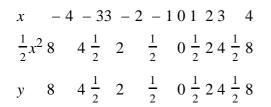
Question:

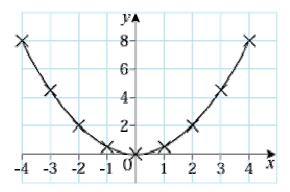
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = \frac{1}{2}x^2$

Solution:

 $y = \frac{1}{2}x^2$





Equation of line of symmetry is x = 0.

Quadratic Equations Exercise A, Question 4

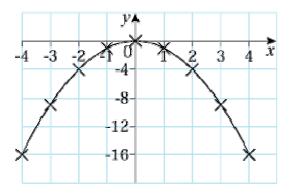
Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = -x^2$

Solution:

 $y = -x^2$



Equation of line of symmetry is x = 0.

Quadratic Equations Exercise A, Question 5

Question:

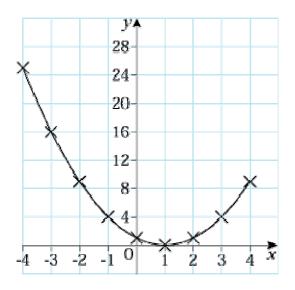
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = (x - 1)^2$

Solution:

 $y = (x-1)^2$

x	- 4	- 33	3 - 2	2 - 1	01234
$(x - 1)^2$	25	16	9	4	10149
у	25	16	9	4	10149



Equation of line of symmetry is x = 1.

Quadratic Equations

Exercise A, Question 6

Question:

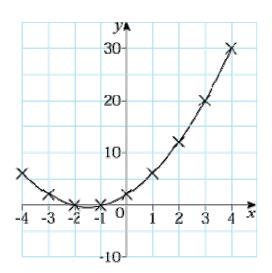
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = x^2 + 3x + 2$

Solution:

 $y = x^2 + 3x + 2$

x	- 4	- 33	- 2	- 1	0	1	2	3	4	
$x^2 + 3x$	+ 2 16 - 12 -	+ 2 9 - 9 +	+ 2 4 - 6 +	21-3-	+ 2 0 + 0	+ 2 1 + 3 +	- 24 + 6	+ 2 9 + 9	+ 2 16 + 12	2 + 2
у	6	2	0	0	2	6	12	20	30	



Equation of line of symmetry is $x = -1 \frac{1}{2}$.

Quadratic Equations Exercise A, Question 7

Question:

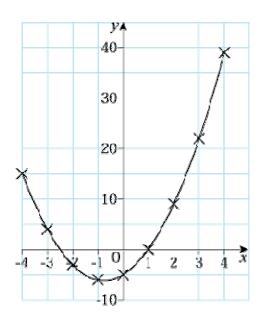
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = 2x^2 + 3x - 5$

Solution:

 $y = 2x^2 + 3x - 5$

x	- 4	- 33	- 2	- 1	0	1	2	3	4	
$2x^2 + 3x$	- 5 32 - 12	- 5 18 - 9 -	58-6-	52-3-	- 5 0 + 0 -	- 5 2 + 3	- 5 8 + 6	- 5 18 + 9	- 5 32 + 1	2 – 5
y	15	4	- 3	- 6	- 5	0	9	22	39	



Equation of line of symmetry is $x = -\frac{3}{4}$.

Quadratic Equations Exercise A, Question 8

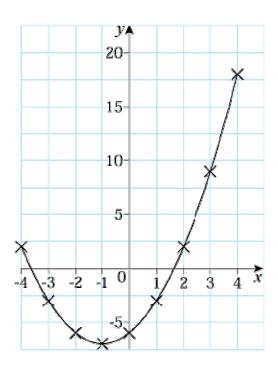
Question:

Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = x^2 + 2x - 6$

Solution:

 $y = x^2 + 2x - 6$



Equation of line of symmetry is x = -1.

Quadratic Equations Exercise A, Question 9

Question:

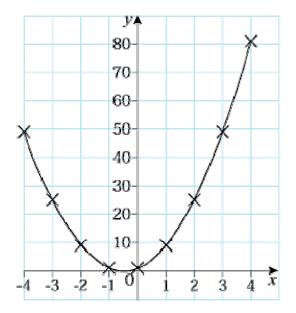
Draw a graph with the following equation, taking values of x from -4 to +4. For each graph write down the equation of the line of symmetry.

 $y = (2x + 1)^2$

Solution:

 $y = (2x+1)^2$

x	- 4	- 33	- 2	- 1	0	1	2	3	4
2x + 1	-8 + 1	-6+1	-4 + 1	-2 + 1	0 + 1	2 + 1	4 + 1	6 + 1	8 + 1
(2x + 1)	-7	-5	-3	-1	1	3	5	7	9
$y = (2x+1)^2$	² 49	25	9	1	1	9	25	49	81



Equation of line of symmetry is $x = -\frac{1}{2}$.

Quadratic Equations Exercise B, Question 1

Question:

Solve the following equation:

 $x^2 = 4x$

Solution:

 $x^{2} - 4x = 0$ x (x - 4) = 0 x = 0 or x - 4 = 0So x = 0 or x = 4

Quadratic Equations Exercise B, Question 2

Question:

Solve the following equation:

 $x^2 = 25x$

Solution:

 $x^{2} - 25x = 0$ x (x - 25) = 0 x = 0 or x - 25 = 0So x = 0 or x = 25

Quadratic Equations Exercise B, Question 3

Question:

Solve the following equation:

 $3x^2 = 6x$

Solution:

 $3x^2 - 6x = 0$ 3x (x - 2) = 0 x = 0 or x - 2 = 0So x = 0 or x = 2

Quadratic Equations Exercise B, Question 4

Question:

Solve the following equation:

 $5x^2 = 30x$

Solution:

 $5x^2 - 30x = 0$ 5x (x - 6) = 0 x = 0 or x - 6 = 0So x = 0 or x = 6

Quadratic Equations Exercise B, Question 5

Question:

Solve the following equation:

 $x^2 + 3x + 2 = 0$

Solution:

(x + 1) (x + 2) = 0 x + 1 = 0 or x + 2 = 0So x = -1 or x = -2

Quadratic Equations Exercise B, Question 6

Question:

Solve the following equation:

 $x^2 + 5x + 4 = 0$

Solution:

(x + 1) (x + 4) = 0 x + 1 = 0 or x + 4 = 0So x = -1 or x = -4

Quadratic Equations Exercise B, Question 7

Question:

Solve the following equation:

 $x^2 + 7x + 10 = 0$

Solution:

(x+2) (x+5) = 0 x+2=0 or x+5=0x=-2 or x=-5

Quadratic Equations Exercise B, Question 8

Question:

Solve the following equation:

 $x^2 - x - 6 = 0$

Solution:

(x-3)(x+2) = 0x-3 = 0 or x+2 = 0So x = 3 or x = -2

Quadratic Equations Exercise B, Question 9

Question:

Solve the following equation:

 $x^2 - 8x + 15 = 0$

Solution:

(x-3)(x-5) = 0 x-3 = 0 or x-5 = 0So x = 3 or x = 5

Quadratic Equations Exercise B, Question 10

Question:

Solve the following equation:

 $x^2 - 9x + 20 = 0$

Solution:

(x-4) (x-5) = 0 x-4 = 0 or x-5 = 0So x = 4 or x = 5

Quadratic Equations Exercise B, Question 11

Question:

Solve the following equation:

 $x^2 - 5x - 6 = 0$

Solution:

(x-6) (x+1) = 0x-6 = 0 or x+1 = 0So x = 6 or x = -1

Quadratic Equations Exercise B, Question 12

Question:

Solve the following equation:

 $x^2 - 4x - 12 = 0$

Solution:

(x-6) (x+2) = 0x-6 = 0 or x+2 = 0So x = 6 or x = -2

Quadratic Equations Exercise B, Question 13

Question:

Solve the following equation:

 $2x^2 + 7x + 3 = 0$

Solution:

(2x + 1) (x + 3) = 0 2x + 1 = 0 or x + 3 = 0 2x = -1 or x = -3So $x = -\frac{1}{2} \text{ or } x = -3$

Quadratic Equations Exercise B, Question 14

Question:

Solve the following equation:

 $6x^2 - 7x - 3 = 0$

Solution:

(3x + 1) (2x - 3) = 0 3x + 1 = 0 or 2x - 3 = 0So $x = -\frac{1}{3} \text{ or } x = \frac{3}{2}$

Quadratic Equations Exercise B, Question 15

Question:

Solve the following equation:

 $6x^2 - 5x - 6 = 0$

Solution:

(3x + 2) (2x - 3) = 0 3x + 2 = 0 or 2x - 3 = 0So $x = -\frac{2}{3} \text{ or } x = \frac{3}{2}$

Quadratic Equations Exercise B, Question 16

Question:

Solve the following equation:

 $4x^2 - 16x + 15 = 0$

Solution:

(2x - 3) (2x - 5) = 0 2x - 3 = 0 or 2x - 5 = 0So $x = \frac{3}{2} \text{ or } x = \frac{5}{2}$

Quadratic Equations Exercise B, Question 17

Question:

Solve the following equation:

 $3x^2 + 5x = 2$

Solution:

 $3x^{2} + 5x - 2 = 0$ (3x - 1) (x + 2) = 0 3x - 1 = 0 or x + 2 = 0 So x = $\frac{1}{3}$ or x = -2

Quadratic Equations Exercise B, Question 18

Question:

Solve the following equation:

 $(2x-3)^2 = 9$

Solution:

 $2x - 3 = \pm 3$ $2x = \pm 3 + 3$ $x = \frac{\pm 3 + 3}{2}$ So x = 3 or x = 0

Quadratic Equations Exercise B, Question 19

Question:

Solve the following equation:

 $(x-7)^2 = 36$

Solution:

 $x - 7 = \pm 6$ $x = \pm 6 + 7$ So x = 1 or x = 13

Quadratic Equations Exercise B, Question 20

Question:

Solve the following equation:

 $2x^2 = 8$

Solution:

 $x^{2} = 4$ $x = \pm 2$ So x = 2 or x = -2

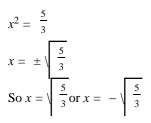
Quadratic Equations Exercise B, Question 21

Question:

Solve the following equation:

 $3x^2 = 5$

Solution:



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Quadratic Equations Exercise B, Question 22

Question:

Solve the following equation:

 $(x-3)^2 = 13$

Solution:

 $\begin{array}{l} x - 3 = \pm \ \sqrt{13} \\ x = 3 \pm \ \sqrt{13} \\ \text{So } x = 3 + \ \sqrt{13} \text{ or } x = 3 - \ \sqrt{13} \end{array}$

Quadratic Equations Exercise B, Question 23

Question:

Solve the following equation:

 $(3x-1)^2 = 11$

Solution:

 $3x - 1 = \pm \sqrt{11}$ $3x = 1 \pm \sqrt{11}$ $x = \frac{1 \pm \sqrt{11}}{3}$

Quadratic Equations Exercise B, Question 24

Question:

Solve the following equation:

 $5x^2 - 10x^2 = -7 + x + x^2$

Solution:

 $-6x^{2} - x + 7 = 0$ (1 - x) (7 + 6x) = 0 x = 1 or 6x = -7 So x = 1 or x = -\frac{7}{6}

Quadratic Equations Exercise B, Question 25

Question:

Solve the following equation:

 $6x^2 - 7 = 11x$

Solution:

 $6x^{2} - 11x - 7 = 0$ (3x - 7) (2x + 1) = 0 3x - 7 = 0 or 2x + 1 = 0 So x = $\frac{7}{3}$ or x = $-\frac{1}{2}$

Quadratic Equations Exercise B, Question 26

Question:

Solve the following equation:

 $4x^2 + 17x = 6x - 2x^2$

Solution:

 $6x^{2} + 11x = 0$ x (6x + 11) = 0 x = 0 or 6x + 11 = 0 So x = 0 or x = - $\frac{11}{6}$

Quadratic Equations Exercise C, Question 1

Question:

Complete the square for the expression:

 $x^2 + 4x$

Solution:

 $= (x+2)^2 - 4$

Quadratic Equations Exercise C, Question 2

Question:

Complete the square for the expression:

 $x^2 - 6x$

Solution:

 $= (x-3)^2 - 9$

Quadratic Equations Exercise C, Question 3

Question:

Complete the square for the expression:

 $x^2 - 16x$

Solution:

 $= (x-8)^2 - 64$

Quadratic Equations Exercise C, Question 4

Question:

Complete the square for the expression:

 $x^{2} + x$

Solution:

$$= \left(\begin{array}{c} x + \frac{1}{2} \end{array} \right)^2 - \frac{1}{4}$$

Quadratic Equations Exercise C, Question 5

Question:

Complete the square for the expression:

 $x^2 - 14x$

Solution:

 $= (x-7)^2 - 49$

Quadratic Equations Exercise C, Question 6

Question:

Complete the square for the expression:

 $2x^2 + 16x$

Solution:

 $= 2 (x^{2} + 8x)$ = 2 [(x + 4)² - 16] = 2 (x + 4)² - 32

Quadratic Equations Exercise C, Question 7

Question:

Complete the square for the expression:

 $3x^2 - 24x$

Solution:

 $= 3 (x^{2} - 8x)$ = 3 [(x - 4)² - 16] = 3 (x - 4)² - 48

Quadratic Equations Exercise C, Question 8

Question:

Complete the square for the expression:

 $2x^2 - 4x$

Solution:

 $= 2 (x^{2} - 2x)$ = 2 [(x - 1)² - 1] = 2 (x - 1)² - 2

Quadratic Equations Exercise C, Question 9

Question:

Complete the square for the expression:

 $5x^2 + 20x$

Solution:

 $= 5 (x^{2} + 4x)$ = 5 [(x + 2)² - 4] = 5 (x + 2)² - 20

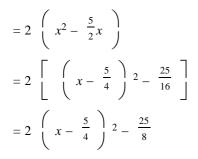
Quadratic Equations Exercise C, Question 10

Question:

Complete the square for the expression:

 $2x^2 - 5x$

Solution:



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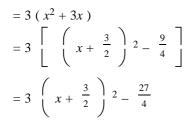
Quadratic Equations Exercise C, Question 11

Question:

Complete the square for the expression:

 $3x^2 + 9x$

Solution:



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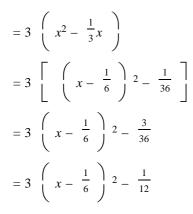
Quadratic Equations Exercise C, Question 12

Question:

Complete the square for the expression:

 $3x^2 - x$

Solution:



Quadratic Equations Exercise D, Question 1

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $x^2 + 6x + 1 = 0$

Solution:

 $x^{2} + 6x = -1$ $(x + 3)^{2} - 9 = -1$ $(x + 3)^{2} = -1 + 9$ $(x + 3)^{2} = 8$ $x + 3 = \pm \sqrt{8}$ $x = -3 \pm \sqrt{8}$ $x = -3 \pm \sqrt{2} \sqrt{4}$ $x = -3 \pm 2\sqrt{2}$ So $x = -3 + 2\sqrt{2}$ or $x = -3 - 2\sqrt{2}$

Quadratic Equations Exercise D, Question 2

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $x^2 + 12x + 3 = 0$

Solution:

 $x^{2} + 12x = -3$ (x + 6)² - 36 = -3 (x + 6)² = 33 x + 6 = ± $\sqrt{33}$ x = -6 ± $\sqrt{33}$ So x = -6 + $\sqrt{33}$ or x = -6 - $\sqrt{33}$

Quadratic Equations Exercise D, Question 3

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $x^2 - 10x = 5$

Solution:

 $(x-5)^{2}-25 = 5$ $(x-5)^{2} = 5 + 25$ $(x-5)^{2} = 30$ $x-5 = \pm \sqrt{30}$ $x = 5 \pm \sqrt{30}$ So $x = 5 + \sqrt{30}$ or $x = 5 - \sqrt{30}$

Quadratic Equations Exercise D, Question 4

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $x^2 + 4x - 2 = 0$

Solution:

 $x^{2} + 4x = 2$ (x + 2)² - 4 = 2 (x + 2)² = 6 x + 2 = ± $\sqrt{6}$ So x = -2 + $\sqrt{6}$ or x = -2 - $\sqrt{6}$

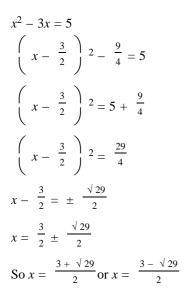
Quadratic Equations Exercise D, Question 5

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $x^2 - 3x - 5 = 0$

Solution:



Quadratic Equations Exercise D, Question 6

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $2x^2 - 7 = 4x$

Solution:

$$2x^{2} - 4x = 7$$

$$x^{2} - 2x = \frac{7}{2}$$

$$(x - 1)^{2} - 1 = \frac{7}{2}$$

$$(x - 1)^{2} = \frac{9}{2}$$

$$x - 1 = \pm \frac{3}{\sqrt{2}}$$

$$x = 1 \pm \frac{3}{\sqrt{2}}$$

$$x = 1 \pm \frac{3\sqrt{2}}{2}$$

Quadratic Equations Exercise D, Question 7

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $4x^2 - x = 8$

Solution:

$$x^{2} - \frac{1}{4}x = 2$$

$$\left(x - \frac{1}{8}\right)^{2} - \frac{1}{64} = 2$$

$$\left(x - \frac{1}{8}\right)^{2} = 2 + \frac{1}{64}$$

$$\left(x - \frac{1}{8}\right)^{2} = \frac{129}{64}$$

$$x - \frac{1}{8} = \pm \frac{\sqrt{129}}{8}$$

$$x = \frac{1}{8} \pm \frac{\sqrt{129}}{8}$$
So $x = \frac{1 + \sqrt{129}}{8}$ or $x = \frac{1 - \sqrt{129}}{8}$

Quadratic Equations Exercise D, Question 8

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $10 = 3x - x^2$

Solution:

$$x^{2} - 3x = -10$$

$$\left(x - \frac{3}{2}\right)^{2} - \frac{9}{4} = -10$$

$$\left(x - \frac{3}{2}\right)^{2} = -\frac{31}{4}$$

No real roots as RHS is negative.

Quadratic Equations Exercise D, Question 9

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $15 - 6x - 2x^2 = 0$

Solution:

 $2x^{2} + 6x = 15$ $x^{2} + 3x = \frac{15}{2}$ $\left(x + \frac{3}{2}\right)^{2} - \frac{9}{4} = \frac{15}{2}$ $\left(x + \frac{3}{2}\right)^{2} = \frac{39}{4}$ $x + \frac{3}{2} = \pm \frac{\sqrt{39}}{2}$ $x = -\frac{3}{2} \pm \frac{\sqrt{39}}{2}$ So $x = -\frac{3}{2} \pm \frac{\sqrt{39}}{2}$ or $x = -\frac{3}{2} - \frac{\sqrt{39}}{2}$

Quadratic Equations Exercise D, Question 10

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

 $5x^2 + 8x - 2 = 0$

Solution:

 $x^{2} + \frac{8}{5}x = \frac{2}{5}$ $\left(x + \frac{4}{5}\right)^{2} - \frac{16}{25} = \frac{2}{5}$ $\left(x + \frac{4}{5}\right)^{2} = \frac{26}{25}$ $x + \frac{4}{5} = \pm \frac{\sqrt{26}}{5}$ $x = -\frac{4}{5} \pm \frac{\sqrt{26}}{5}$ So $x = \frac{-4 + \sqrt{26}}{5}$ or $x = \frac{-4 - \sqrt{26}}{5}$

Quadratic Equations Exercise E, Question 1

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $x^2 + 3x + 1 = 0$

Solution:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$
Then $x = \frac{-3 \pm \sqrt{5}}{2}$ or $x = \frac{-3 - \sqrt{5}}{2}$

Quadratic Equations Exercise E, Question 2

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $x^2 - 3x - 2 = 0$

Solution:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+3 \pm \sqrt{9+8}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$
Then $x = \frac{3 \pm \sqrt{17}}{2}$ or $x = \frac{3 - \sqrt{17}}{2}$

Quadratic Equations Exercise E, Question 3

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $x^2 + 6x + 6 = 0$

Solution:

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2 \times 1}$$

$$x = \frac{-6 \pm \sqrt{36 - 24}}{2}$$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$x = -3 \pm \sqrt{3}$$
Then $x = -3 \pm \sqrt{3}$ or $x = -3 - \sqrt{3}$

Quadratic Equations Exercise E, Question 4

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $x^2 - 5x - 2 = 0$

Solution:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+5 \pm \sqrt{25 + 8}}{2}$$

$$x = \frac{5 \pm \sqrt{33}}{2}$$
Then $x = \frac{5 \pm \sqrt{33}}{2}$ or $x = \frac{5 - \sqrt{33}}{2}$

Quadratic Equations Exercise E, Question 5

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $3x^2 + 10x - 2 = 0$

Solution:

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-2)}}{2 \times 3}$$

$$x = \frac{-10 \pm \sqrt{100 + 24}}{6}$$

$$x = \frac{-10 \pm \sqrt{124}}{6}$$

$$x = \frac{-10 \pm 2\sqrt{31}}{6}$$
Then $x = \frac{-5 \pm \sqrt{31}}{3}$ or $x = \frac{-5 \pm \sqrt{31}}{3}$

Quadratic Equations Exercise E, Question 6

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $4x^2 - 4x - 1 = 0$

Solution:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2 \times 4}$$

$$x = \frac{+4 \pm \sqrt{16 + 16}}{8}$$

$$x = \frac{4 \pm \sqrt{32}}{8}$$

$$x = \frac{4 \pm 4 \sqrt{2}}{8}$$
Then $x = \frac{1 \pm \sqrt{2}}{2}$ or $x = \frac{1 - \sqrt{2}}{2}$

Quadratic Equations Exercise E, Question 7

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $7x^2 + 9x + 1 = 0$

Solution:

$$x = \frac{-9 \pm \sqrt{9^2 - 4(7)(1)}}{2 \times 7}$$

$$x = \frac{-9 \pm \sqrt{81 - 28}}{14}$$

$$x = \frac{-9 \pm \sqrt{53}}{14}$$
Then $x = \frac{-9 \pm \sqrt{53}}{14}$ or $x = \frac{-9 - \sqrt{53}}{14}$

Quadratic Equations Exercise E, Question 8

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $5x^2 + 4x - 3 = 0$

Solution:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-3)}}{2 \times 5}$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{10}$$

$$x = \frac{-4 \pm \sqrt{76}}{10}$$

$$x = \frac{-4 \pm 2\sqrt{19}}{10}$$
Then $x = \frac{-2 \pm \sqrt{19}}{5}$ or $x = \frac{-2 - \sqrt{19}}{5}$

Quadratic Equations Exercise E, Question 9

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $4x^2 - 7x = 2$

Solution:

$$4x^{2} - 7x - 2 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^{2} - 4(4)(-2)}}{2 \times 4}$$

$$x = \frac{+7 \pm \sqrt{49 + 32}}{8}$$

$$x = \frac{7 \pm \sqrt{81}}{8}$$

$$x = \frac{7 \pm 9}{8}$$
Then $x = 2$ or $x = -\frac{1}{4}$

Quadratic Equations Exercise E, Question 10

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

 $11x^2 + 2x - 7 = 0$

Solution:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(11)(-7)}}{2 \times 11}$$

$$x = \frac{-2 \pm \sqrt{4 + 308}}{22}$$

$$x = \frac{-2 \pm \sqrt{312}}{22}$$

$$x = \frac{-2 \pm \sqrt{312}}{22}$$

$$x = \frac{-2 \pm 2\sqrt{78}}{22}$$

$$x = \frac{-1 \pm \sqrt{78}}{11}$$
Then $x = \frac{-1 \pm \sqrt{78}}{11}$ or $x = \frac{-1 - \sqrt{78}}{11}$

Quadratic Equations Exercise F, Question 1

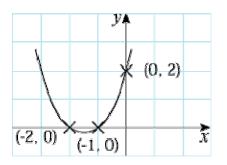
Question:

Sketch the graphs of the following equations:

(a) $y = x^{2} + 3x + 2$ (b) $y = x^{2} - 3x + 10$ (c) $y = x^{2} + 2x - 15$ (d) $y = 2x^{2} + 7x + 3$ (e) $y = 2x^{2} + x - 3$ (f) $y = 6x^{2} - 19x + 10$ (g) $y = 3x^{2} - 2x - 5$ (h) $y = 3x^{2} - 13x$ (i) $y = -x^{2} + 6x + 7$ (j) $y = 4 - 7x - 2x^{2}$

Solution:

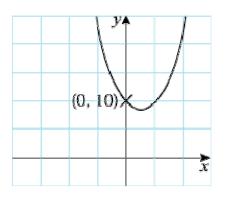
(a) a > 0 so graph is a \cup shape. $b^2 = 9, 4ac = 8$ $b^2 > 4ac$, so there are two different roots of the equation y = 0. When y = 0, (x + 2) (x + 1) = 0 x = -2 or x = -1So crossing points are (-2, 0) and (-1, 0). When x = 0, y = 2, so (0, 2) is a crossing point.



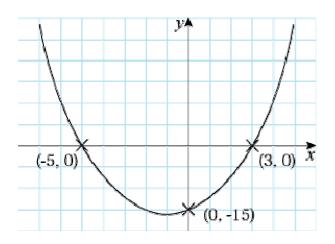
(b) a > 0 so graph is a \cup shape.

 $b^2 = 9, 4ac = 40$

 $b^2 < 4ac$, so there are no real roots of the equation y = 0. So there are no crossing points at y = 0. When x = 0, y = 10, so crossing point is (0, 10).



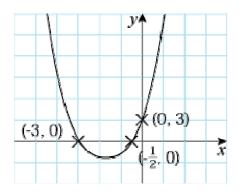
(c) a > 0 so graph is a \cup shape. $b^2 = 4, 4ac = -60$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = (x + 5) (x - 3) x = -5 or x = 3So crossing points are (-5, 0) and (3, 0). When x = 0, y = -15, so crossing point is (0, -15).



(d) a > 0 so graph is a \cup shape. $b^2 = 49, 4ac = 24$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = (2x + 1) (x + 3) $x = -\frac{1}{2}$ or x = -3

So crossing points are $\left(\begin{array}{c} -\frac{1}{2} & 0 \end{array}\right)$ and $\left(\begin{array}{c} -3 & 0 \end{array}\right)$.

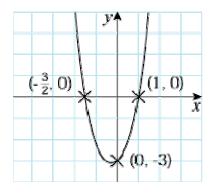
When x = 0, y = 3, so crossing point is (0, 3).



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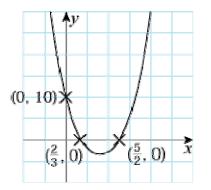
So crossing points are $\left(-\frac{3}{2}, 0 \right)$ and (1, 0).

When x = 0, y = -3, so crossing point is (0, -3).



(f) a > 0 so graph is a \cup shape. $b^2 = 361, 4ac = 240$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = (3x - 2) (2x - 5) $x = \frac{2}{3}$ or $x = \frac{5}{2}$

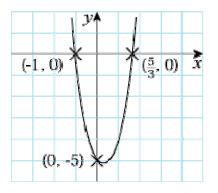
So crossing points are $\left(\frac{2}{3}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$. When x = 0, y = 10, so crossing point is (0, 10).



(g) $a > \text{ so graph is a } \cup \text{ shape.}$ $b^2 = 4, 4ac = -60$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = (3x - 5) (x + 1) $x = \frac{5}{3} \text{ or } x = -1$

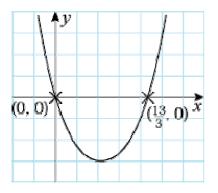
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So crossing points are $\left(\frac{5}{3}, 0\right)$ and (-1, 0). When x = 0, y = -5, so crossing point is (0, -5).

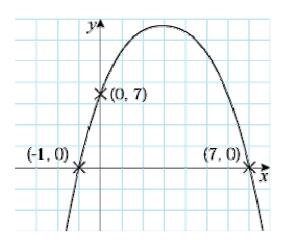


(h) a > 0 so graph is a \cup shape. $b^2 = 169, 4ac = 0$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = x (3x - 13)x = 0 or $x = \frac{13}{3}$

So crossing points are (0, 0) and $\begin{pmatrix} \frac{13}{3}, 0 \end{pmatrix}$. When x = 0, y = 0, so crossing point is (0, 0).



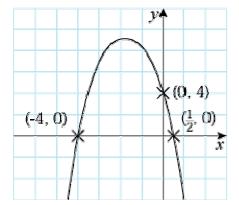
(i) a < 0 so graph is a \cap shape. $b^2 = 36, 4ac = -28$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = (7 - x) (1 + x) x = 7 or x = -1So crossing points are (7, 0) and (-1, 0). When x = 0, y = 7, so crossing point is (0, 7).



(j) a < 0 so graph is a \cap shape. $b^2 = 49, 4ac = -32$ $b^2 > 4ac$, so two different roots of y = 0. When y = 0, 0 = (1 - 2x) (4 + x) $x = \frac{1}{2}$ or x = -4

So crossing points are $\left(\begin{array}{c} \frac{1}{2} \\ 0 \end{array}\right)$ and $\left(\begin{array}{c} -4 \\ 0 \end{array}\right)$.

When x = 0, y = 4, so crossing point is (0, 4).



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Quadratic Equations Exercise F, Question 2

Question:

Find the values of k for which $x^2 + kx + 4 = 0$ has equal roots.

Solution:

 $x^{2} + kx + 4 = 0$ has equal roots if $b^{2} = 4ac$ i.e. $k^{2} = 4 \times 1 \times 4 = 16 \implies k = \pm 4$

Quadratic Equations Exercise F, Question 3

Question:

Find the values of k for which $kx^2 + 8x + k = 0$ has equal roots.

Solution:

 $kx^{2} + 8x + k = 0$ has equal roots if $b^{2} = 4ac$ i.e. $8^{2} = 4 \times k \times k = 4k^{2}$ So $k^{2} = \frac{64}{4} = 16 \implies k = \pm 4$

Quadratic Equations Exercise G, Question 1

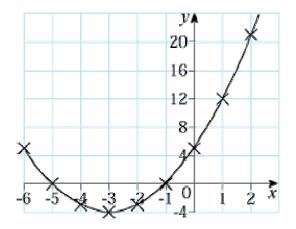
Question:

Draw the graphs with the following equations, choosing appropriate values for x. For each graph write down the equation of the line of symmetry.

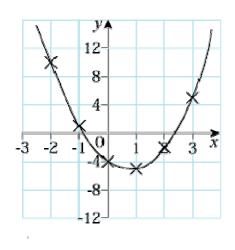
(a) $y = x^2 + 6x + 5$

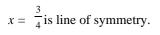
(b) $y = 2x^2 - 3x - 4$

Solution:



x = -3 is line of symmetry.





Quadratic Equations Exercise G, Question 2

Question:

Solve the following equations:

(a) $y^2 + 3y + 2 = 0$

(b) $3x^2 + 13x - 10 = 0$

(c) $5x^2 - 10x = 4x + 3$

(d) $(2x-5)^2 = 7$

Solution:

(a) (y+1) (y+2) = 0y = -1 or y = -2

(b) (3x - 2) (x + 5) = 0 $x = \frac{2}{3}$ or x = -5

(c) $5x^2 - 14x - 3 = 0$ (5x + 1) (x - 3) = 0 $x = -\frac{1}{5}$ or x = 3

(d) $2x - 5 = \pm \sqrt{7}$ $2x = \pm \sqrt{7} + 5$ $x = \frac{5 \pm \sqrt{7}}{2}$

Quadratic Equations Exercise G, Question 3

Question:

Solve the following equations by:

(i) Completing the square.

(ii) Using the formula.

(a) $x^2 + 5x + 2 = 0$

(b) $x^2 - 4x - 3 = 0$

(c) $5x^2 + 3x - 1 = 0$

(d) $3x^2 - 5x = 4$

Solution:

(a) (i)
$$x^{2} + 5x = -2$$

 $\left(x + \frac{5}{2}\right)^{2} - \frac{25}{4} = -2$
 $\left(x + \frac{5}{2}\right)^{2} = \frac{17}{4}$
 $x + \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$
(ii) $x = \frac{-5 \pm \sqrt{17}}{2}$
(iii) $x = \frac{-5 \pm \sqrt{17}}{2}$
 $x = \frac{-5 \pm \sqrt{17}}{2}$
(b)(i) $x^{2} - 4x = 3$
 $(x - 2)^{2} - 4 = 3$
 $(x - 2)^{2} - 4 = 3$
 $(x - 2)^{2} = 7$
 $x = 2 \pm \sqrt{7}$
(ii) $x = \frac{-(-4) \pm \sqrt{16 - 4(1)(-3)}}{2}$
 $x = \frac{+4 \pm \sqrt{16 + 12}}{2}$
 $x = \frac{4 \pm \sqrt{4 \times 7}}{2}$

$$x = \frac{4 \pm 2\sqrt{7}}{2}$$

$$x = 2 \pm \sqrt{7}$$
(c) (i) $5x^{2} + 3x = 1$

$$5\left[\left(x^{2} + \frac{3}{5}x\right)^{2} = 1\right]$$

$$5\left[\left(x + \frac{3}{10}\right)^{2} - \frac{9}{100}\right]^{2} = \frac{9}{100}$$

$$\left(x + \frac{3}{10}\right)^{2} - \frac{9}{100} = \frac{1}{5}$$

$$\left(x + \frac{3}{10}\right)^{2} = \frac{29}{100}$$

$$x + \frac{3}{10} = \pm \frac{\sqrt{29}}{10}$$
(i) $x = \frac{-3 \pm \sqrt{29}}{10}$
(ii) $x = \frac{-3 \pm \sqrt{29}}{10}$
(d)(i) $3\left(x^{2} - \frac{5}{3}x\right) = 4$

$$3\left[\left(x - \frac{5}{6}\right)^{2} - \frac{25}{36}\right] = 4$$

$$\left(x - \frac{5}{6}\right)^{2} - \frac{25}{36} = \frac{4}{3}$$

$$\left(x - \frac{5}{6}\right)^{2} = \frac{73}{36}$$

$$x - \frac{5}{6} = \pm \frac{\sqrt{73}}{6}$$
(ii) $x = \frac{-(-5) \pm \sqrt{25 - 4(3)(-4)}}{6}$

$$x = \frac{+5 \pm \sqrt{25 + 48}}{6}$$

$$x = \frac{5 \pm \sqrt{73}}{6}$$

Quadratic Equations Exercise G, Question 4

Question:

Sketch graphs of the following equations:

(a) $y = x^2 + 5x + 4$

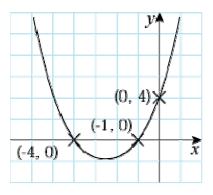
(b) $y = 2x^2 + x - 3$

(c) $y = 6 - 10x - 4x^2$

(d) $y = 15x - 2x^2$

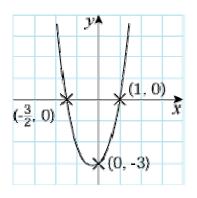
Solution:

(a) a > 0 so \cup shape $b^2 = 25, 4ac = 16$ $b^2 > 4ac$, so two different roots of y = 0. $y = 0 \Rightarrow 0 = (x + 1) (x + 4)$ x = -1 or x = -4So *x*-axis crossing points are (-1, 0) and (-4, 0). $x = 0 \Rightarrow y = 4$ So *y*-axis crossing point is (0, 4).



(b) a > 0 So \cup shape $b^2 = 1, 4ac = -24$ $b^2 > 4ac$, so two different roots of y = 0. $y = 0 \Rightarrow 0 = (2x + 3) (x - 1)$ $x = -\frac{3}{2}$ or x = 1

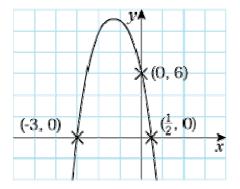
So x-axis crossing points are $\left(-\frac{3}{2}, 0 \right)$ and (1, 0). $x = 0 \Rightarrow y = -3$ so y-axis crossing point in (0, -3).



(c) a < 0 So \cap shape $b^2 = 100, 4ac = -96$ $b^2 > 4ac$, so two different roots of y = 0. $y = 0 \Rightarrow 0 = (1 - 2x) (6 + 2x)$ $x = \frac{1}{2}$ or x = -3

So x-axis crossing points are $\left(\begin{array}{c} \frac{1}{2} \\ 0 \end{array}\right)$ and $\left(\begin{array}{c} -3 \\ 0 \end{array}\right)$.

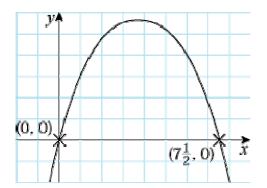
 $x = 0 \Rightarrow y = 6$ so y-axis crossing point is (0, 6).



(d) a < 0 so \cap shape $b^2 = 225, 4ac = 0$ $b^2 > 4ac$, so two different roots of y = 0. $y = 0 \Rightarrow 0 = x (15 - 2x)$ x = 0 or $x = 7\frac{1}{2}$

So *x*-axis crossing points are (0, 0) and $\left(\begin{array}{c} 7\frac{1}{2} \\ 2 \end{array}, 0\right)$.

$$x = 0 \Rightarrow y = 0$$
 So y-axis crossing point is $(0, 0)$.



Quadratic Equations Exercise G, Question 5

Question:

Given that for all values of *x* :

 $3x^2 + 12x + 5 = p(x + q)^2 + r$

(a) Find the values of p, q and r.

(b) Solve the equation $3x^2 + 12x + 5 = 0$. **[E]**

Solution:

(a)
$$3x^2 + 12x + 5 = p(x^2 + 2qx + q^2) + r$$

 $3x^2 + 12x + 5 = px^2 + 2pqx + pq^2 + r$
Comparing $x^2 : p = 3$ (D)
Comparing constants : $pq^2 + r = 5$ (3)
Substitute (D) into (2):
 $2 \times 3q = 12$
 $q = 2$
Substitute $p = 3$ and $q = 2$ into (3):
 $3 \times 2^2 + r = 5$
 $12 + r = 5$
 $r = -7$
So $p = 3, q = 2, r = -7$

(b)
$$3x^2 + 12x + 5 = 0$$

 $\Rightarrow 3(x+2)^2 - 7 = 0$
 $\Rightarrow 3(x+2)^2 = 7$
 $\Rightarrow (x+2)^2 = \frac{7}{3}$
 $\Rightarrow x+2 = \pm \sqrt{\frac{7}{3}}$
So $x = -2 \pm \sqrt{\frac{7}{3}}$

Quadratic Equations Exercise G, Question 6

Question:

Find, as surds, the roots of the equation

 $2(x+1)(x-4) - (x-2)^2 = 0$

Solution:

$$2(x^{2} - 3x - 4) - (x^{2} - 4x + 4) = 0$$

$$2x^{2} - 6x - 8 - x^{2} + 4x - 4 = 0$$

$$x^{2} - 2x - 12 = 0$$

$$x = \frac{-(-2) \pm \sqrt{4 - 4(1)(-12)}}{2}$$

$$x = \frac{+2 \pm \sqrt{52}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 13}}{2}$$

$$x = \frac{2 \pm 2 \sqrt{13}}{2}$$

$$x = 1 \pm \sqrt{13}$$

Quadratic Equations Exercise G, Question 7

Question:

Use algebra to solve (x-1)(x+2) = 18. **[E]**

Solution:

 $x^{2} + x - 2 = 18$ $x^{2} + x - 20 = 0$ (x + 5) (x - 4) = 0 x = -5 or x = 4